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I nvestors always want to earn as much average (mean) return as possible with as little volatility as possible, but in general more return comes with more volatility. Investing is all about deciding what an appropriate risk level is and then getting as much return as possible while only taking the appropriate amount of risk. There are two components to differentiating risk-adjusted returns versus a benchmark. The first component is the weighted average risk-adjusted return of the investments in the portfolio. White Oaks absolutely attempts to select investments that subsequently earn better risk-adjusted returns than the portfolio's benchmark, as it should. The challenge



with this part of the approach is that it typically comes with a compromise in liquidity or a requirement the manager of the investment has some exceptional level of skill that will likely persist in the future. As long as the liquidity of the entire portfolio is managed appropriately, having some illiquidity within a portfolio is not problematic. With regards to evaluating the skill of a manager, White Oaks does go well beyond just evaluating past performance of a manager when assessing a third party manager's skill to mitigate the risk of finding managers who have simply been lucky in the past and are unlikely to see their strong historical performance continue into the future. This paper, however, focuses on a different component to risk-adjusted performance at the portfolio level that does not require that the underlying investments have risk-adjusted performance that is superior to that of the portfolio's benchmark.

This aspect to what White Oaks does is called diversification. Diversification does not by itself increase a portfolio's mean return, but it does reduce a portfolio's volatility. By volatility I mean standard deviation of returns. Standard deviation is a measure of the degree to which returns typically deviate from their average. A low standard deviation means that most annual returns are very close to the average return. A high standard deviation means that it is not uncommon for an annual return to be significantly higher



or significantly lower than the average. Typically in investing, asset classes with higher standard deviations also have higher mean returns. Stocks, for example, tend to have a higher level of standard deviation than bonds and are thus viewed as riskier than bonds. As compensation for taking that extra risk, investors in stocks are typically rewarded with higher mean returns over time.

Diversification offers investors a way to reduce portfolio standard deviation without reducing portfolio mean return. The secret to this trick is correlation. Correlation is a statistical measure of the degree to which two data series (in this case returns to two different asset classes) move together. Correlation ranges between -1 and 1. A correlation of -1 means the worst returns for one asset class occur at the same time as the best returns for the other asset class, mediocre returns for one asset class occur at the same time as pretty good returns for the other asset class, and average returns for both asset classes occur at the same time as the best returns for the second asset class, the worst returns for the first asset class occur at exactly the same time as the worst returns for the second asset class, and average returns for the first asset class occur at exactly the same time as the worst returns for the second asset class, and average returns for both asset classes occur at the same time as the worst returns for the second asset class, and average returns for the first asset class occur at exactly the same time as the worst returns for the second asset class, and average returns for both asset classes occur at the same time. A correlation of 0 means that the returns of the first asset class tell you nothing about what the returns of the second asset class likely were.

A simple example may help illustrate how diversification works. Chart 1 shows the performance of two assets, Asset A and Asset B, and the performance of a portfolio that allocates equally to both assets with continuous rebalancing. Asset A delivers a return of 25% in the first year and Asset B delivers a return of -5% in the first year. In the second year, Asset A is -5% and Asset B is 25%. This two-year cycle repeats itself five times, so ultimately Asset A and Asset B grow a \$100 investment to \$236.14 over the course of ten years, meaning both Asset A and Asset B deliver an 8.97% return per year. The equal-weighted portfolio also grows \$100 to \$236.14 over ten years, so its annualized mean return is also 8.97%. In this example, mean returns are clearly unaffected by diversification.

Volatility, on the other hand, is significantly impacted by diversification. Imagine if the portfolio were just 100% allocated to Asset A or to Asset B. In some years the portfolio would be up 25% and in other years it would be down -5%. Annualized standard deviation would be 14.46% for either Asset A or Asset B as stand-alone portfolios. This would be a much less comfortable experience than the equal-weighted portfolio would provide.

The equal-weighted portfolio would be up 8.97% every year with an annualized standard deviation of 0%. There would be no really big up years and no down years. The reason there is no volatility in the equal-weighted portfolio is because every time one asset has a bad year, that bad year is offset by a good year from the other asset. In the real world, things are not quite this simple. There are more than two assets and the returns are less predictable and not perfectly negatively correlated with each other. The concept still works in the real world though. Diversifying may not eliminate volatility entirely like in Chart 1's example, but it does reduce it.

Chart 1



**WARNING:** Readers who are math lovers will enjoy seeing how the mathematics of diversification works in the next few paragraphs, but those who are less quantitatively inclined may want to just skim this section and then skip down to the section titled "**An Extreme Example**."

# **Mathematics of Diversification**

The average return of a portfolio is simply the weighted average return of its underlying components. The standard deviation of a portfolio's return is at most the weighted average standard deviation of its components and, in most cases, is even less than that. The only situation where a portfolio's standard deviation is as high as the weighted average standard deviation of the underlying components is when the correlations between all those components are 1 (i.e. - a perfect positive correlation). If there is only one component, this will obviously be the case. Such a scenario suggests that there is no diversification at all. Any level of diversification will reduce portfolio standard deviation and will mean the portfolio's risk-adjusted return will be better than the weighted average risk-adjusted return of the portfolio's components. The lower the correlation between components, the more portfolio standard deviation is reduced relative to the weighted average standard deviation of a portfolio's components.

A simple two-asset portfolio example illustrates this nicely. Assume the mean return to asset A is 10% per year and the mean return to asset B is 10% per year. Assume asset A has a standard deviation of 15% per year and asset B has a standard deviation of 17% per year. Assume the correlation between



asset A and asset B is 0.3 and each asset has a 50% allocation in the portfolio. The portfolio's mean return would be 10% (0.5\*0.1 + 0.5\*0.1 = 0.1). The portfolio's standard deviation would be  $12.91\% ([0.5^2*0.15^2+0.5^2*0.17^2+2*0.3*0.5*0.5*0.15*0.17]^0.5 = 0.1291)$ . Notice that the weighted average standard deviation in the portfolio is 16% (0.5\*0.15 + 0.5\*0.17 = 0.16), so diversification reduces volatility by 3.09% (0.16 - 0.1291 = 0.0309) in this case. Throughout this paper I will refer to the amount by which standard deviation is reduced because of diversification (3.09% in this example) as the diversification benefit.

Equations for portfolio means and standard deviations are provided in Equation 1 (two asset portfolio), Equation 2 (three asset portfolio), and Equation 3 (n asset portfolio).

### **Equation 1**

Equations for portfolio means and standard deviations when there are only two assets (asset A and asset B) are as follows:

 $\mu_p = w_a \times \mu_a + w_b \times \mu_b$ 

$$\sigma_p = (w_a^2 \times \sigma_a^2 + w_b^2 \times \sigma_b^2 + 2 \times \rho \times w_a \times w_b \times \sigma_a \times \sigma_b)^{0.5}$$

Where

 $\mu_p$  = portfolio mean return

 $w_a$  = weight of asset A

 $\mu_a$  = mean return of asset A

 $w_b$  = weight of asset B

 $\mu_b$  = mean return of asset B

 $\sigma_p$  = portfolio standard deviation

 $\sigma_a$  = standard deviation of asset A

 $\sigma_b$  = standard deviation of asset B

 $\rho$  = correlation between asset A and asset B

### **Equation 2**

With three assets (asset A, asset B, and asset C), portfolio means and standard deviations are as follows:



 $\mu_p = \mathbf{w}_a \times \mu_a + \mathbf{w}_b \times \mu_b + \mathbf{w}_c \times \mu_c$ 

 $\sigma_{p} = (w_{a}^{2} \times \sigma_{a}^{2} + w_{b}^{2} \times \sigma_{b}^{2} + w_{c}^{2} \times \sigma_{c}^{2} + 2 \times \rho_{a,b} \times w_{a} \times w_{b} \times \sigma_{a} \times \sigma_{b} + 2 \times \rho_{a,c} \times w_{a} \times w_{c} \times \sigma_{a} \times \sigma_{c} + 2 \times \rho_{b,c} \times w_{b} \times w_{c} \times \sigma_{b} \times \sigma_{c})^{0.5}$ 

Where

- $\mu_p$  = portfolio mean return
- $w_a$  = weight of asset A
- $\mu_a$  = mean return of asset A
- w<sub>b</sub> = weight of asset B
- $\mu_b$  = mean return of asset B
- $w_c$  = weight of asset C
- $\mu_c$  = mean return of asset C
- $\sigma_p$  = portfolio standard deviation
- $\sigma_a$  = standard deviation of asset A
- $\sigma_b$  = standard deviation of asset B
- $\sigma_c$  = standard deviation of asset C
- $\rho_{a,b}$  = correlation between asset A and asset B
- $\rho_{a,c}$  = correlation between asset A and asset C
- $\rho_{b,c}$  = correlation between asset B and asset C

### **Equation 3**

The portfolio mean and standard deviation can be generalized to n assets using the following equations.

 $\mu_p = \mathbf{w} \cdot \boldsymbol{\mu}$ 

 $\sigma_p = (\mathbf{w} \cdot \mathbf{C} \cdot \mathbf{w}^{\mathsf{T}})^{\mathsf{0.5}}$ 



Where

- $\mu_p$  = portfolio mean return
- $\sigma_p$  = portfolio standard deviation
- n = number of assets in portfolio
- w = 1×n vector of weights of the different assets in portfolio
- $\mu = n \times 1$  vector of mean returns of the different assets in portfolio
- C = n×n covariance matrix of assets in portfolio

### **An Extreme Example**

The neat thing about diversification is that it does not require a compromise on liquidity and it does not require the investor to be an expert at predicting the future. As long as correlations between asset classes are less than perfectly positive and there are several asset classes in the portfolio, risk can be reduced significantly without reducing mean return.

As an extreme example, assume you have a portfolio with 100 different asset classes. Every asset class has an expected return of 10% per year and a standard deviation of 24% per year. As an aside, these are White Oaks' prospective assumptions for emerging market stocks, so this portfolio has 100 different asset classes that all have the risk and return profiles of emerging market stocks. This portfolio is clearly allocated exclusively to investments that would be considered by most investors to be very risky. Of course, if we assume these 100 asset classes are all completely uncorrelated to each other, something very interesting happens. This portfolio, which is invested exclusively in very risky investments, only has an annualized standard deviation of 2.4%. For reference, White Oaks is currently assuming an annualized standard deviation of 3% for bonds, which most investors would consider to be conservative. Through the magic of diversification, a portfolio that only has investments with emerging markets equities' type risk has less than bond like risk because it is extremely well diversified.

Of course, the bond investor is likely to earn much lower portfolio mean returns (White Oaks currently assumes 2.5%) than the 100-asset investor, whose portfolio mean return is 10%. The reason the 100-asset investor can reasonably expect higher mean returns is because he is taking more risk with each investment and is being compensated for that additional risk with more expected return, but at the portfolio level most of that risk is being diversified away. To assume that investments with more volatility do not generally offer higher mean returns is to make an assumption about differences in the Sharpe Ratio across different asset classes, which is a different part of the decomposition of portfolio risk-adjusted returns.



This 100-asset investor gets emerging equity type mean returns at bond-like risk by fully exploiting the power of diversification. Chart 2 shows the portfolio standard deviation of other portfolios that exclusively invest in assets with standard deviations of 24% per year and shows varying numbers of investments and average correlations between those investments. Zero correlation and 100 hundred assets takes annualized portfolio standard deviation from 24% to 2.40%. Even a 0.4 correlation and only ten assets takes annualized portfolio standard deviation from 24% to 16.28%. With a zero correlation and five assets, annualized portfolio standard deviation goes from 24% to 10.73%.

Chart 2 gives a nice illustration of how much more effective diversification is when correlations are lower.





# White Oaks Pooled Fund Portfolios

Thus far, the discussion has only centered around diversification benefits for theoretical portfolios. This section of the paper applies the math to actual portfolios White Oaks is managing for clients compared to their benchmarks. All numbers are prospective rather than actual numbers historically experienced over one specific period of time, but the assumptions for each asset class are largely based upon historical

data. We are certainly happy to discuss any of these assumptions in more detail with anyone who is interested.

Table 1 shows assumed mean returns and standard deviations for the asset classes used in this section's analysis.

#### Table 1

Annualized Mean Ann. Standard Deviation **Domestic Equities** 6.0% 16.0% Int'l Dev. Equities 9.0% 17.0% 10.0% Emerging Mkt Equities 24.0% Bonds 2.5% 3.0% 5.5% Trade Finance 0.4% Small Business Lending 11.0% 2.0% 0.1% 0.1% Cash Commodities 4.0% 12.0% Twin Win 5.0% 13.0% Gold 7.5% 18.0% MLPs 12.0% 16.0% WhiteBox 12.0% 10.0% 5.0% 12.0% Managed Futures Equity Long/Short 10.0% 9.0% MBS 3.7% 3.0% Anchor 9.0% 2.0% Market Neutral (Eq. L/S) 10.0% 16.0%

10.0%

10.0%

12.0%

4.0%

18.0%

5.0%

Private Equity

Traded REITs

Non-traded REITs



Table 2 shows portfolio allocations for the White Oaks pooled funds, including the Freedom Fund, which is a new fund that launched at the beginning of the second quarter of 2016. These are target allocations as of December 2016.

	Aggressive	Flexible	Alternative	Moderate	Low Volatility	Freedom
Domestic Equities	39.0%	34.0%	38.0%	38.0%	27.0%	0.0%
Int'l Dev. Equities	14.0%	12.0%	14.0%	14.0%	10.0%	0.0%
Emerging Mkt Equities	4.0%	3.0%	3.0%	4.0%	3.0%	0.0%
Bonds	0.0%	0.0%	5.0%	5.0%	10.0%	0.0%
Trade Finance	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Small Business Lending	4.7%	5.9%	4.3%	4.5%	5.5%	10.0%
Cash	0.0%	0.0%	0.0%	0.0%	0.0%	5.0%
Commodities	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Twin Win	1.2%	1.4%	1.1%	0.9%	1.4%	2.5%
Gold	2.2%	2.5%	2.0%	1.6%	2.5%	4.6%
MLPs	3.4%	4.4%	3.2%	3.1%	4.2%	7.4%
WhiteBox	2.1%	2.7%	1.9%	2.0%	2.5%	4.4%
Managed Futures	4.6%	5.4%	4.2%	3.5%	5.4%	9.6%
Equity Long/Short	2.9%	3.3%	2.6%	2.1%	3.3%	5.9%
MBS	2.3%	3.0%	2.0%	2.2%	2.7%	4.9%
Anchor	4.9%	6.3%	4.4%	4.6%	5.8%	10.5%
Market Neutral (Eq. L/S)	4.7%	6.1%	4.3%	4.5%	5.7%	10.2%
Private Equity	0.0%	0.0%	0.0%	0.0%	0.0%	10.0%
Traded REITs	3.5%	3.5%	3.5%	2.4%	3.9%	5.3%
Non-traded REITs	6.5%	6.5%	6.5%	7.6%	7.1%	9.7%
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

#### Table 2



Table 3 shows the assumed correlation matrix of the different asset classes used.



#### Table 3



In Charts 3-8, I show prospective returns and standard deviations for the different White Oaks pooled funds as well as for commonly referenced stock and bond mixes, which use domestic equities and bonds as the only two asset classes. As a reminder, the scatter points that are farther to the right have a higher standard deviation, which is inferior to lower standard deviation (points on the left side) since most people are assumed to be risk averse. Scatter points that are higher have higher mean returns, which is superior to lower mean returns. In Charts 3-8, I show each portfolio's prospective mean return and prospective standard deviation, but I also show each portfolio's stand-alone risk (SAR). The SAR is just the weighted average standard deviation of the asset classes in the portfolio. The difference between the portfolio standard deviation and the SAR measures how much risk reduction is accomplished due to diversification, which I call the diversification benefit. Taking the 100-asset portfolio with all assets that have 24% standard deviations and zero correlations with each other as an example, the SAR would be 24%, the standard deviation would be 2.4%, and the diversification benefit would be 21.8%. This portfolio is an extreme example of allocating to several high volatility (and hopefully high return as compensation for the high volatility) assets that are uncorrelated with each other.









### Chart 4















believes to be reasonable. These numbers are in no way indicative of actual past or future performance. Please see assumption page attached for more detail.



### Chart 7











The increase in slope from the Aggressive SAR to the Aggressive portfolio, shows how much improvement in risk–adjusted return can be attributed to diversification alone. Notice there is no difference in mean return (shown on the vertical axis in Chart 3) between Aggressive SAR and Aggressive. Aggressive just has less volatility (shown on the horizontal axis in Chart 3) than Aggressive SAR because it incorporates the amount of volatility reduction that comes from diversification. The same can be said of the other White Oaks portfolios shown in Charts 4-8. Charts 3-8 also show the 80/20 and 80/20 SAR and the 60/40 and 60/40 SAR even though there are no lines to highlight the change in slope from 80/20 SAR to 80/20 and from 60/40 SAR to 60/40. There is still some improvement in risk-adjusted returns from diversification benefits for these benchmarks as well.

If I apply the assumptions for the actual portfolios and benchmarks to the chart shown in Chart 2, these are the resulting numbers. The Bonds benchmark has one asset, so the average correlation is 1 and the assumed standard deviation is 3.0%. The Stocks benchmark also has one asset with an average correlation of 1 and a standard deviation of 16.0%. In both Bonds and Stocks, the SAR equals the standard deviation because there is no diversification benefit due to both only having one asset. The 60/40 benchmark has two assets, an SAR of 10.8%, an implied average correlation of 0.6, a standard deviation of 9.6%, and a diversification benefit of 1.2% (SAR minus standard deviation). The 80/20 benchmark has two assets, an SAR of 13.4%, an implied average correlation of 0.8, a standard deviation of 12.8%, and a diversification benefit of 0.6%.

Notice that the 60/40 and 80/20 benchmarks both hold the same two assets, domestic stocks and bonds. The implied average correlation is different between them though. This is because the average correlation calculation assumes equal weighting and equal standard deviation for all assets. If one asset has a heavier allocation and higher standard deviation, and thus is consuming more than its equal share of the risk budget, the implied average correlation will be higher because part of the implied average correlation with itself.

Equation 4 shows the equation for the implied average correlation calculation.

### Equation 4

$$\rho = [(\sigma_p^2 - ((1/N)^2)x(S^2)xN] / [((1/N)^2)x(S^2)x(Nx(N-1))]$$

Where

 $\rho$  = implied average correlation between all N assets

N = number of assets in portfolio

 $\sigma_p$  = portfolio standard deviation

#### S = weighted average standard deviation on all N assets in portfolio

The Aggressive Fund has fifteen assets, an SAR 13.7%, an implied average correlation of 0.6, a standard deviation of 10.5%, and a diversification benefit of 3.2%. The Flexible Fund has fifteen assets, an SAR of 13.0%, an implied average correlation of 0.5, a standard deviation of 9.4%, and a diversification benefit



of 3.6%. The Alternative Fund has sixteen assets, an SAR of 13.2%, an implied average correlation of 0.6, a standard deviation of 10.1%, and a diversification benefit of 3.1%. The Moderate Fund has sixteen assets, an SAR of 13.1%, an implied average correlation of 0.6, a standard deviation of 10.1%, and a diversification benefit of 3.0%. The Low Volatility Fund has sixteen assets, an SAR of 11.8%, an implied average correlation of 8.1%, and a diversification benefit of 3.7%. The Freedom Fund has fourteen assets, an SAR of 8.5%, an implied average correlation of 0.2, a standard deviation of 4.1%, and a diversification benefit of 4.4%. The 100-asset hypothetical portfolio from Chart 2 has 100 assets, an SAR of 24.0%, an implied average correlation of 0, a standard deviation of 2.4%, and a diversification benefit of 21.6%. Table 4 summarizes this information.

Table 4											
			Implied								
			Number of	Average	CAD	Standard	Diversification				
			Assels	Correlation	ЗАК	Deviation	Benefit				
Benchmarks	Bonds		1	1.0	3.0%	3.0%	0.0%				
	Stocks		1	1.0	16.0%	16.0%	0.0%				
	60/40		2	0.6	10.8%	9.6%	1.2%				
	80/20		2	0.8	13.4%	12.8%	0.6%				
White Oaks	Aggressive		15	0.6	13.7%	10.5%	3.2%				
	Flexible		15	0.5	13.0%	9.4%	3.6%				
	Alternative		16	0.6	13.2%	10.1%	3.1%				
	Moderate		16	0.6	13.1%	10.1%	3.0%				
	Low Volatility		16	0.4	11.8%	8.1%	3.7%				
	Freedom		14	0.2	8.5%	4.1%	4.4%				
Hypothetical	100-Asset		100	0.0	24.0%	2.4%	21.6%				

This table clearly indicates that, at least based upon the assumptions, the White Oaks portfolios experience more reduction in volatility due to diversification than traditional stock/bond portfolios based upon the diversification benefit numbers. The table also indicates that even though White Oaks is enjoying more diversification benefits than its benchmarks, the hypothetical 100-asset portfolio shows that there is still plenty of room for White Oaks to continue improving its asset class diversification.

Continuing to increase the diversification benefit by adding more uncorrelated assets, would reduce the volatility of the White Oaks portfolios. It would not necessarily increase the expected return of the White Oaks portfolios. More leverage or a higher allocation to higher mean return and higher volatility assets (both so as not to assume newly introduced assets could necessarily deliver a higher risk-adjusted return) could increase expected returns. The combination of these two effects can potentially increase expected returns and decrease volatility without the need for newly introduced assets that have higher risk-adjusted returns than the currently held ones.

Chart 9 shows a nice visual on the impact of diversification and then leverage.



# Conclusion

Allocating to many different uncorrelated asset classes reduces portfolio volatility via diversification benefits and, thus, does not necessarily translate into less expected portfolio return accompanying less portfolio volatility the way allocating more to lower volatility/lower expected return investments would. White Oaks already takes advantage of the mathematics of diversification more than its benchmarks, and the room for improvement that Table 4 shows is possible in terms of the size of 100-Asset portfolio's diversification benefit is exciting in terms of the opportunity it presents for a manager like White Oaks that understands diversification better than most. The alternative asset landscape is ripe with a diverse set of opportunities with which White Oaks can continue to improve its diversification and continue to



leave its benchmarks and many other managers whose focus is solely on traditional asset classes, at a disadvantage.

#### About the Author

Alex joined White Oaks Investment Management in April 2014 after spending nearly 8 years doing investment research at The Roseline Financial Group in Richmond, Virginia. As an Investment Analyst at White Oaks, he performs investment research, asset allocation of various strategies, assesses client portfolios and assists with overall client communication.

Alex holds a Bachelor of Arts in Economics and a Minor in Mathematics from the College of William and Mary in Williamsburg, Virginia and a Master of Science in Finance from the University of Illinois at Urbana-Champaign, where he specialized in financial engineering. He is also a CFA Charterholder.